Summary of Effect Sizes and their Links to Inferential Statistics

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Much of this is based on:

Please feel free to contact me if any of these formulae seem incorrect – it is possible that typographical errors may have been made.
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1. EFFECT SIZES: DEFINITIONS

1.1 Degree of association between two variables (Correlational Effect Sizes)

\[ r = \Phi = r_{pb} = \frac{\sum Z_X Z_Y}{n} = \frac{\sum \left( \frac{X - \bar{X}}{\sqrt{\sum (X - \bar{X})^2}} \right) \left( \frac{Y - \bar{Y}}{\sqrt{\sum (Y - \bar{Y})^2}} \right)}{n} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\frac{n}{\sigma_x \sigma_y} = \frac{s_{xy}}{\sigma_x \sigma_y}} \]

NOTE: Some of these formulas use \( n \) in the denominator of the correlation, but they are sometimes written with \( n-1 \) rather than \( n \) in the denominators. This difference does not matter as long as either \( n \) or \( n-1 \) is used in all parts of the equation (ie, in the standard deviations, z-scores, covariance, etc).

When \( r \) is a point-biserial correlation (\( r_{pb} \)), it is based on a dichotomous grouping variable (\( X \)) and a continuous variable (\( Y \)). In this case, the equation can also be written as:

\[ r = \sqrt{p_1 p_2 \left( \bar{Y}_1 - \bar{Y}_2 \right)} \]

where \( p_1 \) and \( p_2 \) are the proportions of the total sample in each group, \( \left( \bar{Y}_1 - \bar{Y}_2 \right) \) is the difference between the groups’ means on the continuous variable (\( Y \)), and \( \sigma_y \) is the standard deviation of variable \( Y \). Note that this is Equation 5 in McGrath and Meyer (2006), and note that the direction of the correlation depends on which group is considered group 1 and which is considered group 2.


Z transformation of a correlation

\[ z_r = r' = \frac{1}{2} \log_e \left[ \frac{1+r}{1-r} \right] = \frac{1}{2} \left[ \log_e (1 + r) - \log_e (1 - r) \right] \]

Where \( \log_e \) is the natural log function (LN on some calculators)

To transform back from \( z_r \) (\( r' \)) metric to \( r \) metric

\[ r = \frac{e^{2z_r} - 1}{e^{2z_r} + 1} \]

Where \( e \) is the exponent function (e\(^x\) on some calculators)

Effect size for the difference between correlations

Cohen’s \( q = Z_{r1} - Z_{r2} \)
1.2 **Degree of difference between two means (Effect sizes a la d)**

1.2.1. For comparing means from two groups:

Cohen’s $d = \frac{\bar{X}_1 - \bar{X}_2}{\sigma_{\text{pooled}}}$

Hedges’s $g = \frac{\bar{X}_1 - \bar{X}_2}{s_{\text{pooled}}}$

Glass’s $\Delta = \frac{\bar{X}_1 - \bar{X}_2}{s_{\text{control group}}}$

Where

\[ \sigma_{\text{pooled}} = \sqrt{\frac{(n_1)\sigma_1^2 + (n_2)\sigma_2^2}{n_1 + n_2}} \quad \text{and} \quad s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \]

and

\[ \sigma_{\text{pooled}} = s_{\text{pooled}} \sqrt{\frac{n_1 + n_2 - 2}{n_1 + n_2}} \]

and

\[ s_{\text{pooled}} = \frac{\sigma_{\text{pooled}}}{\sqrt{\frac{n_1 + n_2 - 2}{n_1 + n_2}}} = \frac{s_{\text{pooled}}}{\sqrt{\frac{N - 2}{N}}} \]

You may also see $\sigma_{\text{pooled}}$ referred to as $\sigma_{\text{within}}$, and $s_{\text{pooled}}$ referred to as $s_{\text{within}}$ or as $\sqrt{\text{MS}_{\text{within}}}$

1.2.2 The logic of $d$ and $g$ can be applied when comparing one mean to a population mean (e.g., one sample t-test), such that:

\[ d = \frac{\bar{X} - \mu}{\sigma_X} \]

\[ g = \frac{\bar{X} - \mu}{s_X} \]

where $\mu$ is the null hypothesis population mean

1.2.3 The logic of $d$ and $g$ can be applied when comparing two correlated means (e.g., repeated measures t-test, paired samples t-test)

\[ d = \frac{\bar{D}}{\sigma_D} \]

\[ g = \frac{\bar{D}}{s_D} \]

where $\bar{D}$ is the mean difference score and $\sigma_D$ and $s_D$ are standard deviations of the difference scores
1.3 “Variance accounted for” $R^2$, Eta squared ($\eta^2$), and omega squared ($\omega^2$)

$$R^2 = \eta^2 = \frac{SS_{EXPLAINED}}{SS_{TOTAL}} = \frac{SS_{BETWEEN}}{SS_{TOTAL}}$$

For a specific effect (i.e., in a study with multiple IVs/predictors) $R^2 = \eta^2 = R^2_{\text{EFFECT}} = \eta^2_{\text{EFFECT}} = \frac{SS_{\text{EFFECT}}}{SS_{TOTAL}}$

Omega squared for an effect $= \omega^2_{\text{EFFECT}} = \frac{\sigma^2_{\text{EFFECT}}}{\sigma^2_{\text{TOTAL}}} = \frac{SS_{\text{EFFECT}} - df_{\text{EFFECT}}MS_{\text{ERROR}}}{SS_{TOTAL} + MS_{\text{ERROR}}}$

1.4 Effect sizes for proportions

Cohen’s $g = p - .50$

where $p$ estimates a population proportion

$d' = p_1 - p_2$

where $p_1$ and $p_2$ are estimates of the population proportions

Cohen’s $h = \arcsin p_1 - \arcsin p_2$

Pr obit $d' = Z_{p_1} - Z_{p_2}$

where $Z_{p_1}$ and $Z_{p_2}$ are standard normal deviate transformed estimates of population proportions

Logit $d' = \log_e \left[ \frac{p_1}{1-p_1} \right] - \log_e \left[ \frac{p_2}{1-p_2} \right]$
2. TRANSFORMING BETWEEN EFFECT SIZES

2.1 Computing $r$

2.1.1 Computing $r$ from Cohen’s $d$

For one group and for two correlated means (ie repeated measures or paired samples)

$$r = \sqrt{\frac{d^2}{d^2 + 1}} = \frac{d}{\sqrt{d^2 + 1}}$$

For two independent groups

$$r = \sqrt{\frac{d^2}{d^2 + \frac{1}{p_1 p_2}}}$$

Where $p_1$ is the proportion of participants who are in Group 1 and $p_2$ is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $r = \sqrt{\frac{d^2}{d^2 + 4}}$

2.1.2. Computing $r$ from Hedge’s $g$

For one group and for two correlated means (ie repeated measures or paired samples)

$$r = \sqrt{\frac{g^2}{g^2 + df/N}} = \sqrt{\frac{g^2}{g^2 + N - 1/N}}$$

For two independent groups

$$r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df} = \sqrt{\frac{g^2 + df/N p_1 p_2}{g^2 + N - 2/N p_1 p_2}}}$$

Where $p_1$ is the proportion of participants who are in Group 1 and $p_2$ is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $r = \sqrt{\frac{g^2}{g^2 + 4(N - 2/N)}}$
2.2 Computing d

2.2.1 Computing d from r

For one group and for two correlated means (ie repeated measures or paired samples)

\[ d = \frac{r}{\sqrt{1 - r^2}} \]

For two independent groups

\[ d = \frac{r}{\sqrt{1 - r^2}} \left( \frac{p_1 p_2}{1 - r^2} \right) \]

Where \( p_1 \) is the proportion of participants who are in Group 1 and \( p_2 \) is the proportion in Group 2

Note, for equal sample sizes (\( p_1 = p_2 = .50 \)), this simplifies to:

\[ d = \frac{2r}{\sqrt{1 - r^2}} \]

2.2.2 Computing d from Hedge’s g

For one group and for two correlated means (ie repeated measures or paired samples)

\[ d = g \sqrt{\frac{N}{\text{df}}} = g \sqrt{\frac{N}{N-1}} \]

For two independent groups (regardless of the relative sample sizes)

\[ d = g \sqrt{\frac{N}{\text{df}}} = g \sqrt{\frac{N}{N-2}} \]

2.3 Computing g

2.3.1 Computing g from r

For one group and for two correlated means (ie repeated measures or paired samples)

\[ g = \frac{r}{\sqrt{1 - r^2}} \sqrt{\frac{\text{df}}{N}} \frac{\sqrt{N-1}}{\sqrt{N}} \]

For two independent groups

\[ g = \frac{r}{\sqrt{p_1 p_2 \left(1 - r^2\right)}} \sqrt{\frac{\text{df}}{N}} \frac{\sqrt{N-2}}{\sqrt{N}} \]

Where \( p_1 \) is the proportion of participants who are in Group 1 and \( p_2 \) is the proportion in Group 2

Note, for equal sample sizes (\( p_1 = p_2 = .50 \)), this simplifies to:

\[ g = \frac{2r}{\sqrt{1 - r^2}} \sqrt{\frac{\text{df}}{N}} = \frac{2r}{\sqrt{1 - r^2}} \sqrt{\frac{N-2}{N}} \]
2.3.2 Computing $g$ from Cohen’s $d$

For one group and for two correlated means (i.e., repeated measures or paired samples)

$$g = d \sqrt{\frac{df}{N}} = d \sqrt{\frac{N-1}{N}}$$

For two independent groups (regardless of the relative sample sizes)

$$g = d \sqrt{\frac{df}{N}} = d \sqrt{\frac{N-2}{N}}$$

2.4 Transforming between eta squared ($\eta^2$) and omega squared ($\omega^2$)

$\eta^2$ for an effect $= \eta^2 = \frac{df_{effect} + \frac{nko^2}{1-\omega^2}}{\frac{1}{df_{error}}} \left( \frac{1}{1-\omega^2} \right) + 1$

Where $n$ is the number of individuals per group, and $k$ is the number of groups for the effect.

$\omega^2$ for an effect $= \omega^2 = \frac{\frac{df_{error} \eta^2}{1-\eta^2} - df_{effect}}{\left( \frac{df_{error} \eta^2}{1-\eta^2} - df_{effect} \right) + nk}$
3. COMPUTING SIGNIFICANCE TESTS FROM EFFECT SIZES

Recall, \[ \text{Inferential test statistic} = \text{Effect size} \times \text{Size of Study} \]

3.1 For a 2 x 2 Contingency table

\[ \chi^2 = Z^2 = r^2 N \]

3.2 For a t-test

3.2.1 T from r

This is appropriate for any kind of t-test:

\[ t = \frac{r}{\sqrt{1 - r^2}} \sqrt{df} \]

3.2.2 T from Cohen’s d

3.2.2.1 For a One-sample t-test or correlated means t-test

\[ t = d \sqrt{df} = d \sqrt{N - 1} \quad \text{where} \quad d = \frac{\bar{X} - \mu}{\sigma_X} \quad \text{or} \quad d = \frac{\bar{D}}{\sigma_D} \]

3.2.2.2 For an independent groups t-test

\[ t = d \sqrt{\frac{n_1 n_2}{(n_1 + n_2)}} \sqrt{df} = d \sqrt{P_1 P_2 df} = d \sqrt{P_1 P_2 (N - 2)} \]

Where \( P_1 \) is the proportion of participants who are in Group 1 and \( P_2 \) is the proportion in Group 2.

Note, for equal sample sizes (\( P_1 = P_2 = .50 \)), this simplifies to:

\[ t = d \sqrt{\frac{df}{2}} = d \sqrt{\frac{N - 2}{2}} \]

3.2.3 T from Hedge’s g

3.2.3.1 For a One-sample t-test or correlated means t-test

\[ t = g \sqrt{N} \quad \text{where} \quad g = \frac{\bar{X} - \mu}{s_X} \quad \text{or} \quad g = \frac{\bar{D}}{s_D} \]

3.2.3.2 For an independent groups t-test

\[ t = g \sqrt{\frac{n_1 n_2}{n_1 + n_2}} = g \sqrt{P_1 P_2 N} \]

Where \( P_1 \) is the proportion of participants who are in Group 1 and \( P_2 \) is the proportion in Group 2.

Note, for equal sample sizes (\( n_1 = n_2 = n \) and \( P_1 = P_2 = .50 \)), this simplifies to:

\[ t = g \sqrt{\frac{N}{2}} = g \sqrt{\frac{n}{2}} \]

3.3 For an ANOVA (F test)
3.3.1 For an ANOVA with \( \text{df}_{\text{NUMERATOR}} = 1 \) (two independent groups)

\[
F = \frac{r^2}{1-r^2} \left( \text{df}_{\text{error}} \right)
\]

\[
F = d^2 \left( \text{df}_{\text{error}p_1 p_2} \right)
\]

(for equal n study, \( F = d^2 \left( \frac{\text{df}_{\text{error}}}{4} \right) \))

\[
F = g^2 \left( \frac{n_1 n_2}{n_1 + n_2} \right) = g^2 (nkp_1 p_2) = g^2 (Np_1 p_2) \text{ for a one-way ANOVA}
\]

(for equal n study, \( F = g^2 \left( \frac{nk}{4} \right) = g^2 \left( \frac{N}{4} \right) \) for a one-way ANOVA)

\[
F = \frac{\eta^2}{1-\eta^2} \left( \text{df}_{\text{error}} \right)
\]

\[
F = \frac{\omega^2}{1-\omega^2} (nk)+1 = \frac{\omega^2}{1-\omega^2} (N)+1 \text{ for a one-way ANOVA}
\]

3.3.2 For an ANOVA with \( \text{df}_{\text{NUMERATOR}} > 1 \) (more than two independent groups)

\[
F = \frac{\eta^2}{1-\eta^2} \left( \frac{\text{df}_{\text{error}}}{\text{df}_{\text{means}}} \right)
\]

\[
F = \frac{\omega^2}{1-\omega^2} \left( \frac{nk}{k-1} \right)+1
\]
4. COMPUTING EFFECT SIZES FROM SIGNIFICANCE TESTS

4.1 Computing \( r \)

4.1.1 \( r \) from a \( X^2 \) test for a 2x2 contingency table

\[
r = \Phi = r_{pb} = \sqrt{\frac{\chi^2}{n}} = \frac{Z}{\sqrt{n}}
\]

4.1.2 \( r \) from any t test (or F-test with numerator df = 1)

\[
r = \sqrt{\frac{t^2}{t^2 + df}} = \sqrt{\frac{F}{F + df_{error}}}
\]

4.2 Computing \( d \)

4.2.1 \( d \) from a one-sample t-test or correlated means t-test

\[
d = \frac{t}{\sqrt{df}} = \frac{t}{\sqrt{N - 1}}
\]

4.2.2 \( d \) from an independent groups t-test

\[
d = t \left( \frac{n_1 + n_2}{\sqrt{df \cdot n_1 \cdot n_2}} \right) = \frac{t}{\sqrt{p_1 \cdot p_2 \cdot df_{error}}} = \frac{t}{\sqrt{p_1 \cdot p_2 \cdot (N - 2)}}
\]

Which simplifies to \( d = \frac{2t}{\sqrt{df}} \) if the groups have equal \( n \)

4.2.3 \( d \) from an F test based on numerator df = 1

\[
d = \frac{\sqrt{F}}{\sqrt{p_1 \cdot p_2 \cdot df_{error}}} = \frac{\sqrt{F}}{\sqrt{p_1 \cdot p_2 \cdot (N - 2)}}
\]

Which simplifies to \( d = \frac{2\sqrt{F}}{\sqrt{df}} \) if the groups have equal \( n \)
4.3 Computing $g$

4.3.1 $g$ from a one-sample t-test or correlated means t-test

$$g = \frac{t}{\sqrt{N}}$$

4.3.2 $g$ from an independent groups t-test

$$g = t \sqrt{\frac{n_1 + n_2}{n_1 n_2}} = \frac{t}{\sqrt{p_1 p_2 N}}$$

Which simplifies to $g = \frac{2t}{\sqrt{N}}$ if the groups have equal $n$.

4.3.3 $g$ from an F test based on numerator df = 1

$$g = \frac{\sqrt{F}}{\sqrt{p_1 p_2 N}}$$

Which simplifies to $g = \frac{2\sqrt{F}}{\sqrt{N}}$ if the groups have equal $n$.

4.4 Computing $\eta^2$ and $\omega^2$

$\eta^2$ for an effect = $\eta^2 = \frac{F_{\text{effect}} (df_{\text{effect}})}{F_{\text{effect}} (df_{\text{effect}}) + df_{\text{error}}} = \frac{F_{\text{effect}} (df_{\text{effect}})}{F_{\text{effect}} (df_{\text{effect}}) + df_{\text{error}}}$

$\omega^2$ for an effect = $\omega^2 = \frac{(F_{\text{effect}} - 1)(df_{\text{effect}})}{(F_{\text{effect}} - 1)(df_{\text{effect}}) + nk}$

For F tests with numerator = 1, these simplify to

$\eta^2$ for an effect = $\eta^2 = \frac{F_{\text{effect}}}{F_{\text{effect}} + df_{\text{error}}}$

$\omega^2$ for an effect = $\omega^2 = \frac{(F_{\text{effect}} - 1)}{(F_{\text{effect}} - 1) + nk}$